4. (a) Express the following matix as the sum of a symmetric and a skew-symmetric matrix :

$$\begin{bmatrix} -1 & 7 & 1 \\ 2 & 3 & 4 \\ 5 & 0 & 5 \end{bmatrix}.$$

(b) Prove that every orthogonal matrix is non-singular.

Unit III

5. (a) Using Maclaurin's series, expand $\tan x$ upto the term containing x^5.

(b) Find the radius of curvature of the point $(3a/2, 3a/2)$ of the curve $x^3 + y^3 = 3axy$.

6. (a) Prove that :

$$\beta(m, n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)}$$

(b) Find the coordinates of the centre of curvature at any point of the parabola $y^2 = 4ax$. Hence show that its evolute is $27ay^2 = 4(x-2a)^3$.

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B. Tech. EXAMINATION, 2024

(First Semester)

(C-Scheme) (Re-appear Only)

(CSE)

MATH101C

MATHEMATICS-I

Time : 3 Hours] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **9** is compulsory. All questions carry equal marks.

Unit I

1. (a) Prove that :

$$A^3 - 4A^2 - 3A - 11I = 0$$

where $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -1 \\ 1 & 2 & 3 \end{bmatrix}$.

(b) Reduce the following matrix into its normal form and hence find its rank.

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

2. (a) Solve with the help of determinants, the equations :

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

(b) Apply Gauss elimiantion method to solve the equations :

$$x + 4y - z = -5$$
$$x + y - 6z = -12$$
$$3x - y - z = 4$$

Unit II

3. (a) Find the eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

(b) Find the characteristic equation of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

and hence find its inverse.

Compulsory Question

9. (a) If A and B are symmetric matrices, prove that AB is symmetric iff AB = BA.

(b) Define inner product spaces.

(c) Find the eigen values of the matrix :

$$A = \begin{bmatrix} a & h & g \\ 0 & b & f \\ 0 & 0 & c \end{bmatrix}$$

(d) State Taylor's theorem with remainders.

(e) Define basis of a vector space.

(f) State rank-nullity theorem.

Unit IV

7. (a) Express the matrix $A = \begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$ in the vector space of 2×2 matrices as a linear combination of $B = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$.

(b) Show that the set $\{(2, 1, 4), (1, -1, 2), (3, 1, -2)\}$ forms a basis of R.

8. (a) Find the linear transformation $T : R^3 \rightarrow R^3$ whose range space is spanned by the vectors $(1, 2, 3)$, $(4, 5, 6)$.

(b) Find the matrix representing the transformation $T : R^2 \rightarrow R^2$ given by :

$$T(x_1, x_2) = (3x_1 - x_2, 2x_1 + 4x_2, 5x_1 - 6x_2)$$

relative to the standard basis of R^2 and R^3.